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DYNAMIC METHOD OF DETERMINING AERODYNAMIC CHARACTERISTICS OF MODELS FROM ON EXPERIMENTAL RESULTS OBTAINED IN SHORT-DURATION WIND TUNNELS

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A technique is developed to recover the loads acting in model testing in short-duration wind tunnels. The model together with the load-measuring device is assumed to be a linear dynamic system. Normal responses of the system to unit loads are determined experimentally. The solution of a system of integral Volterra equations of the first kind is found in the class of piecewise-constant functions. Examples are given on solving a test problem with a known exact solution and a problem of determining the aerodynamic characteristics of the HB-2 *reference model on the basis of loads obtained in wind-tunnel measurements.*

Key words: *wind tunnel, strain-gauging, reconstruction of loads.*

Introduction. The following methods are used for processing the measured results if a strain-gauge balance is applied to measure time-dependent forces and moments acting on the model being tested in short-duration wind tunnels: 1) averaging technique; 2) analytical method; 3) statistical method; 4) simplified statistical method [1]. The averaging technique requires a fairly long time of wind-tunnel operation, so that oscillations caused by initial shock loads could decay, and a large time interval with unchanged flow parameters. Even if these conditions are satisfied, however, there are some uncontrollable systematic errors. Three other methods involve the principle of inertia compensation. For this purpose, accelerations are measured at certain points by accelerometers. The methods are based on the following assumptions: the model is a rigid body; the oscillating system consists of the model and a certain part of the balance to be determined; the products of the velocities of revolution of the system around its axes are rather small and can be neglected, as compared with angular accelerations.

In the analytical method, the main difficulty is to establish the portion of the system participating in motion and subsequently determine the mass, center of mass, and matrix of the moments of inertia. The accuracy or reconstructing the acting aerodynamic loads also depends on the coordinates of points where the accelerometers are mounted.

The statistical method is based on linearization of relations between the inertia forces and accelerations measured by accelerometers with the use of the following additional assumption: the aerodynamic coefficients are constant over a chosen time interval determining the operation regime of the wind tunnel. The number of equations (6) is smaller than the number of unknowns (42). As the system of equations is valid at an arbitrary time during the run, however, it is solved on a certain interval by the least squares technique.

The simplified statistical method implies a linear dependence between the acceleration and the corresponding force, which reduces the number of unknowns. Three accelerometers are used, one in each direction. The measurement axes coincide with the directions of the body-fitted coordinate system. The unknown coefficients of proportionality depend on the geometric and mass characteristics of the model, which have to be determined in each test for the model mounted in the test section of the wind tunnel. The system is excited by a pulse impactor,

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which induces system oscillations around the zero line without the action of aerodynamic forces. From the measured accelerations of the sting and balance measurements of forces and moments, one can find coefficients used subsequently to find the aerodynamic characteristics on a certain interval.

The solution obtained by statistical methods significantly depends on the position and length of the time interval. The assumption on constant aerodynamic coefficients can be invalid on some part of the interval length necessary to solve the problem. It is also necessary to correlate the periods of system oscillations with the time interval of a steady flow, which imposes additional restrictions on the geometric and mass characteristics of the model.

1. Dynamic Method. The present paper, which continues the research [2], describes a technique that takes into account the dynamics of the model and the fact that the flow parameters are time-dependent. This technique is devoid of drawbacks indicated above. It is assumed that the responses of the strain-gauge balance registered in time are described by ordinary differential equations with constant coefficients, i.e., the system consisting of the model, sting, strain-gauge balance, and fixturing system is a linear dynamic object. This assumption is based on the following circumstances: the acting loads are such that the relation between strains and stresses is described by Hooke's law; the data of the strain-gauge balance weakly depend on the distribution of loads acting on the model if the integral values of loads are fixed (this is validated, e.g., by the results of testing the model of an air-breathing engine and its elements in a blowdown wind tunnel in flows with Mach numbers M $= 2$, 4, and 6 with simultaneous measurements of forces and moments by mechanical and strain-gauge balances [3, 4]); the size of the sensors is small as compared with the characteristic size of the model; the change in temperature of the model and sensors during the experiment is insignificant. In this case, the response of the system to unit loads being measured experimentally, a system of integral equations is derived to recover the acting loads from the responses of the strain-gauge balance measured in time.

1.1. *Initial Equations.* Let the balance responses $y_j(t)$ in the measurement channels satisfy a system of ordinary differential equations with constant coefficients of the form

$$\sum_{j=1}^{n} \varphi_{ij}\Big(\frac{d}{dt}\Big)\, y_j(t) = f_i(t) \qquad (i = \overline{1, n}, \quad n \leqslant 6), \tag{1.1}$$

where $\varphi_{ij}(d/dt)$ are polynomials of the differentiation operator d/dt and $f_i(t)$ are the external loads such that $f_i(t) = 0$ for $t \leqslant 0$. As is known, if the values of the sought functions and their corresponding derivatives equal zero at $t = 0$, the solution of system (1.1) is the normal response to the external load, and the solution of the homogeneous system is absent from the general solution. Such a situation corresponds to test conditions in short-duration wind tunnels. In this case, the solution of system (1.1) has the form

$$y_j(t) = -\sum_{k=1}^{n} \int_0^t \frac{\partial U_{jk}(t-\tau)}{\partial \tau}\, f_k(\tau)\, d\tau, \qquad j = \overline{1, n}, \tag{1.2}$$

where $U_{jk}(t)$ is the normal response of the jth component to the unit load over the kth component. In (1.2), all quantities have the zeroth dimension, except for time, which can be taken with an arbitrary scale. In the program implementation, the time is normalized to the discretization step h.

1.2. *Determination of Normal Responses.* We introduce the matrices of the generalized static loads $G = \{G_{ji}\}$ (G_{ji} is the value of the jth component in the ith variant of loading), balance readings $Y(t) = \{y_{ji}(t)\}$, coefficients of the influence of the static loads over the kth components on the readings of the jth components $W = \{W_{jk}\}$, and normal responses $U(t) = \{U_{jk}(t)\}$ ($i = \overline{1, m}$, $j, k = \overline{1, n}$, $m \geqslant n$, where n is the number of components and m is the number of loads). The matrix W is found from the matrix equation

$$Y_0 = WG, \tag{1.3}$$

where Y_0 are the steady measured responses of the strain-gauge balance, $(GG')^{-1}$ is the nondegenerate matrix, and G' is the transposed matrix. Normalization is performed so that the diagonal elements of the matrix W are equal to unity: $W_{jj} = 1$.

To determine the functions $U_{jk}(t)$ experimentally, one can reasonably use the unloading method. The model is first loaded by a generalized force [force and (or) moment of the force in a certain inertial coordinate system]. After that, the load is released within a small time interval δt, and the output signals $Y(t)$ are recorded. The initial loaded state is described by relation (3.1), and the process after unloading (the force $-G = \text{const}$ acts at the time $t = 0$) is described by the equation

$$Y(t) = Y_0 - U(t)\, G. \tag{1.4}$$

From Eq. (1.4), one can determine the matrix of the normal responses

$$U(t) = W - (Y(t)G')(GG')^{-1}. \tag{1.5}$$

The number of loads is normally greater than the number of strain-gauge balance components used, which allows one to determine a combination of loads with the condition number $\mu \approx 1$ of the matrix G^{-1}. Assuming that $m = n$, we use Eq. (1.5) to obtain

$$U(t) = W - Y(t)\, G^{-1}. \tag{1.6}$$

As the system is stable, we have $\lim\limits_{t\to\infty} Y(t) = 0$, and Eq. (1.6) yields $\lim\limits_{t\to\infty} U(t) = W$. This circumstance is used to control the accuracy of dynamic calibrations. In reality, the measurements are performed with a discrete time step h determined by the instrumentation used.

2. Determination of Forces and Moments in the Class of Piecewise-Constant Functions. The problem of solving the system of integral equations (1.2) in the general case is ill-posed [5]. In the method described below, problem regularization is achieved by constructing the solution in the class of piecewise-constant functions on a set of finite segments $[t_i, t_{i+1}]$ that are united into the total interval where the solution is determined $[0, t_e]$ [t_e is the final (end) time of the process] under the condition that the equations are satisfied on the average in each interval.

The initial equations (1.2) are written in the vector-matrix form

$$\boldsymbol{Y}(t) = -\int\limits_0^t \frac{\partial U(t-\tau)}{\partial \tau}\, \boldsymbol{f}(\tau)\, d\tau. \tag{2.1}$$

Here $\boldsymbol{Y}(t)$ is the vector function with the components $y_j(t)$, $j = \overline{1,n}$. Let the solutions $\{\boldsymbol{f}_l\}$ be determined on the segments $[t_l, t_{l+1}]$, where $l = \overline{0, i-1}$. Then, the residual function $\boldsymbol{z}(t)$ [difference between the measured $\boldsymbol{Y}_*(t)$ and computed $\boldsymbol{Y}(t)$ values of the signals of the strain-gauge balance] for the ith interval is

$$\begin{gathered} \boldsymbol{z}(t) = \boldsymbol{Q}(t) - U(t-t_i)\boldsymbol{f}_i, \qquad t \in [t_i, t_{i+1}], \\ \boldsymbol{Q}(t) = \boldsymbol{Y}_*(t) + \sum_{l=0}^{i-1} [U(t-t_{l+1}) - U(t-t_l)]\boldsymbol{f}_l. \end{gathered} \tag{2.2}$$

As $\boldsymbol{U}(0) = d\boldsymbol{U}(0)/dt = 0$, the minimum length of the interval should be a finite quantity. Using the fact that the residue over the interval equals zero on the average and assuming that $\boldsymbol{f}_i = \text{const}$, we perform averaging in Eq. (2.2):

$$\langle \boldsymbol{z}_i \rangle = \frac{1}{\Delta t_i} \int\limits_{t_i}^{t_{i+1}} \boldsymbol{z}(t)\, dt, \qquad \langle \boldsymbol{Q}_i \rangle = \frac{1}{\Delta t_i} \int\limits_{t_i}^{t_{i+1}} \boldsymbol{Q}(t)\, dt, \qquad \langle U_i \rangle = \frac{1}{\Delta t_i} \int\limits_{t_i}^{t_{i+1}} U(t-t_i)\, dt.$$

Assuming that $\langle \boldsymbol{z}_i \rangle = 0$, we obtain a solution of Eq. (2.2):

$$\begin{gathered} \boldsymbol{f}_i = \langle U_i \rangle^{-1} \langle \boldsymbol{Q}_i \rangle, \\ \langle \boldsymbol{Q}_i \rangle = \frac{1}{\Delta t_i} \int\limits_{t_i}^{t_{i+1}} \boldsymbol{Y}_*(t)\, dt + \frac{1}{\Delta t_i} \sum_{l=0}^{i-1} \Big[\int\limits_{t_i}^{t_{i+1}} U(t-t_{l+1})\, dt - \int\limits_{t_i}^{t_{i+1}} U(t-t_l)\, dt \Big] \boldsymbol{f}_l. \end{gathered} \tag{2.3}$$

The sought solution depends on the number of intervals N and on the distribution of their lengths, i.e., on the function $\Delta t(i)$ (i is the number of the interval). To obtain this distribution, we introduce the functional of the root-mean-square residue

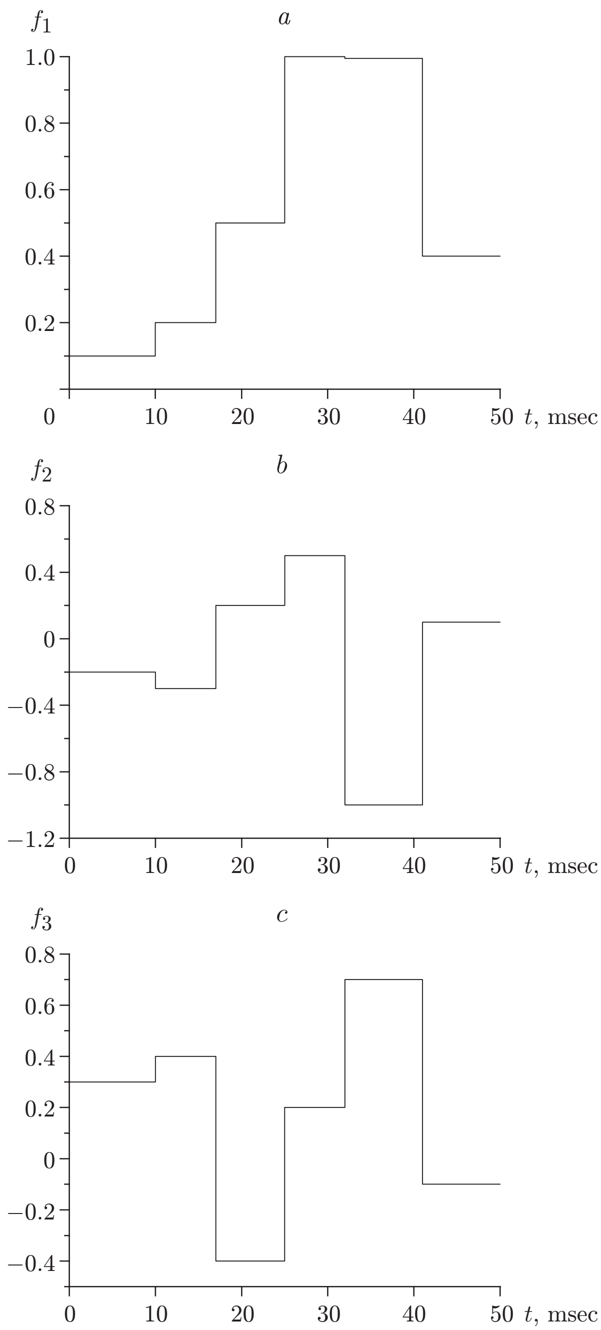

Fig. 1

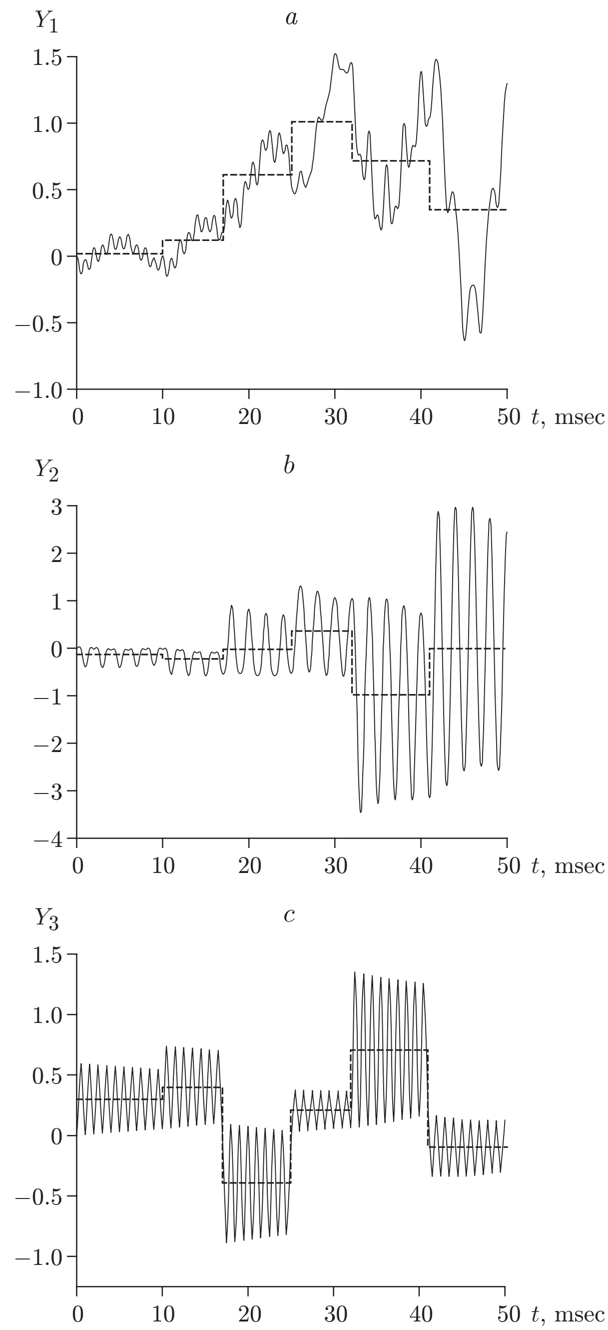

Fig. 2

Fig. 1. Influence of the "force" $\boldsymbol{f}$ over three input channels in the model problem.

Fig. 2. Responses of the model object to the action of the "force" $\boldsymbol{f}$ (solid curves) and the mean values of the responses over the intervals (dashed curves).

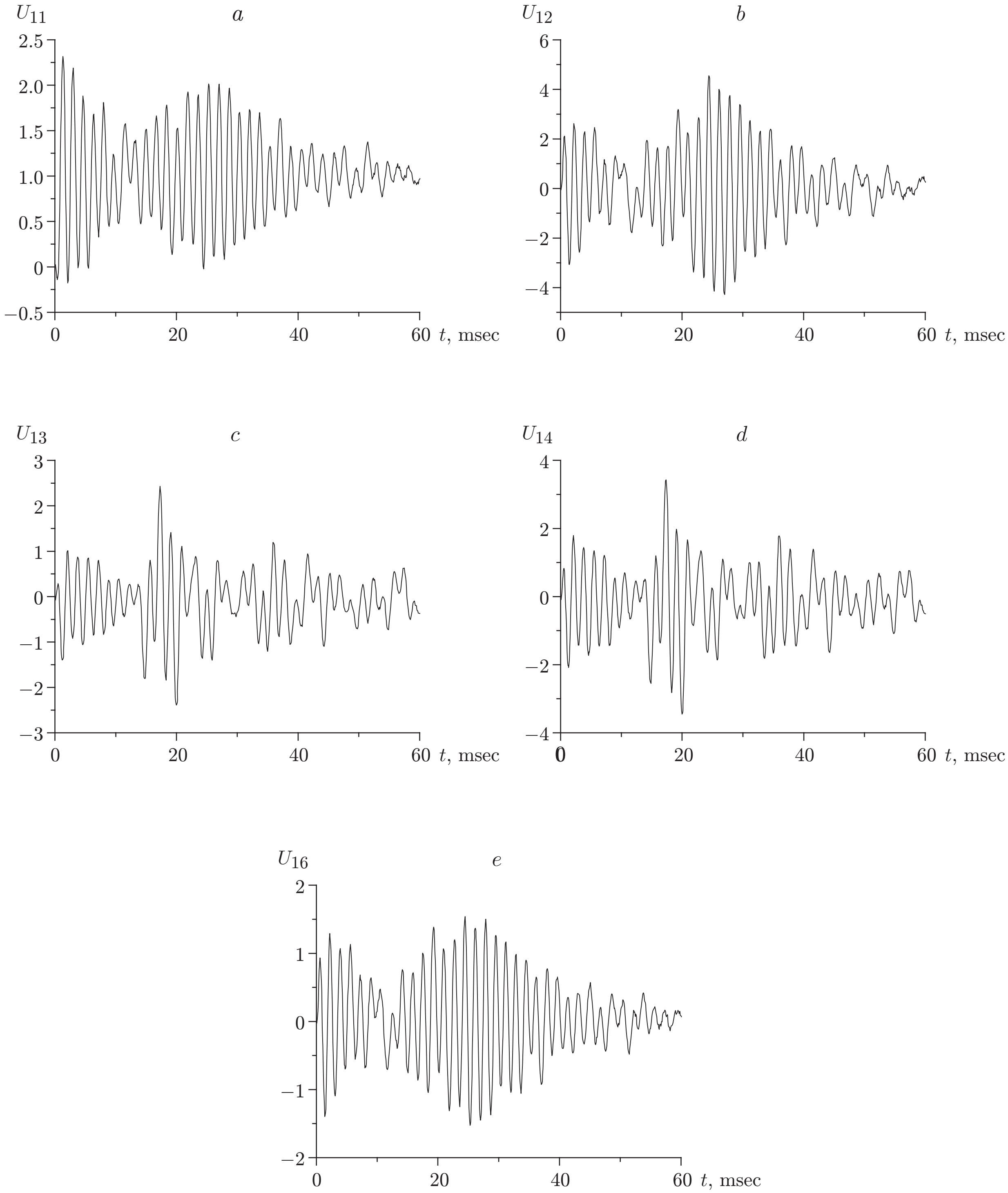

Fig. 3. Normal responses in terms of the f_x component to unit loads of the HB-2 reference model: forces f_x (a), f_y (b), and f_z (c); moments m_x (d) and m_z (e).

$$\Phi(N, \Delta t(i)) = \frac{1}{t_e} \int\limits_0^{t_e} \boldsymbol{z}'(t) \cdot \boldsymbol{z}(t)\, dt. \tag{2.4}$$

In minimizing functional (2.4), the arguments are used to determine the number of intervals, the distribution of their lengths, and the sought solution (2.3). It follows from the computed results that functional (2.4) has many local minimums. This circumstance has to be taken into account in choosing the minimization method. In the present paper, we use the method of coordinate scanning of the functional at a prescribed number of points with choosing

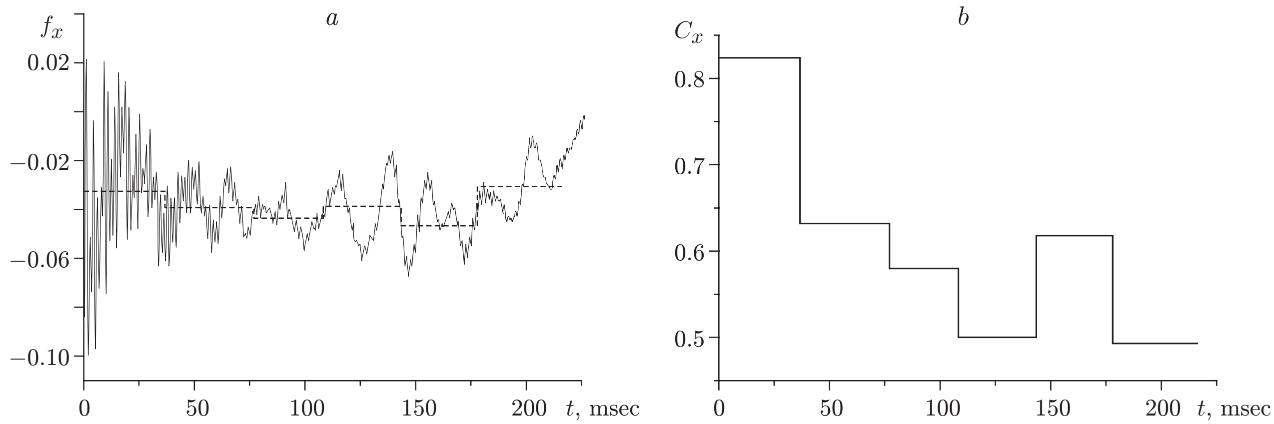

Fig. 4. Experimental values of the longitudinal force f_x (a) and the corresponding drag coefficient C_x (b) versus time for the HB-2 model for M $= 12$ and $\alpha = 12°$: the dashed curve shows the mean values of the longitudinal force over the intervals.

the best point. In the general case, the algorithm does not allow obtaining the global minimum of the functional because it requires an extremely large amount of computations. As compared with algorithms of searching for a local extremum, however, this algorithm predicts significantly lower values of functional (2.4).

Comment 1. The proposed technique of reconstructing time-dependent forces acting on the model does not impose any restrictions on the model mass.

Comment 2. Requirements to the quality of design and fabrication of the strain-gauge balance in terms of minimization of mutual effects can be less rigorous, because the technique takes into account the mutual effects of the recorded signals in all channels.

Comment 3. As the sensors of the strain-gauge balance are deformed under arbitrary loads, a six-component strain-gauge balance should be used even if the information is obtained for generalized forces with the number of components smaller than six.

3. Model Problem. To illustrate the algorithm operation, we consider a linear dynamic model object with three actions and three responses to be recorded. We define the matrix of the normal responses

$$U(\tau) = W(1 - \mathrm{e}^{-\tau}(\beta^{-1}\sin\beta\tau + \cos\beta\tau)), \qquad \beta = 2\pi\omega_0 t_e, \quad t = \tau t_e \tag{3.1}$$

($t_e = 50$ msec) and the matrices ω_0 [msec^{-1}] and W that describe the frequencies of the mutual influence over the channels and the limiting values of the normal responses as $\tau \to \infty$ (analog of the static calibration matrix):

$$\boldsymbol{\omega}_0 = \begin{pmatrix} 0.1 & 0.5 & 1.0 \\ 0.1 & 0.5 & 1.0 \\ 0.1 & 0.5 & 1.0 \end{pmatrix}, \qquad W = \begin{pmatrix} 1.0 & 0.1 & -0.2 \\ -0.2 & 1.0 & 0.3 \\ 10^{-2} & 10^{-3} & 1.0 \end{pmatrix}. \tag{3.2}$$

In addition, six intervals and piecewise-constants "forces" $\boldsymbol{f}$ in these intervals are set (Fig. 1). On a segment $[0, t_e]$ with a step $h = 0.04$ msec, we use Eq. (2.1) with allowance for (3.1) and (3.2) to compute the response of the object to the action of the "force" $\boldsymbol{f}$ (Fig. 2). These values are further used as the measurement data to recover the "forces." Figure 2 shows the computed mean responses in the intervals determined in solving the problem. The error of the solution in terms of the functional and intervals is approximately 10^{-11}. The accuracy of the solution depends on noise intensity and on the law of its distribution. In the problem of determining the aerodynamic characteristics of the HB-2 reference model, noise is present in the form of measurement errors.

4. HB-2 Reference Model. Figure 3 shows the normal responses over the streamwise coordinate to unit loads in five channels for the HB-2 reference model, which has normalized aerodynamic characteristics. The model was mounted in the test section of the AT-303 wind tunnel of the Khristianovich Institute of Theoretical and Applied Mechanics (ITAM), Siberian Division, Russian Academy of Sciences. Within the time interval $t \in [0.6–70]$ msec of the wind-tunnel operation regime, the loads in all channels exert a significant effect on the readings of the drag channel. A similar picture is observed for other channels as well.

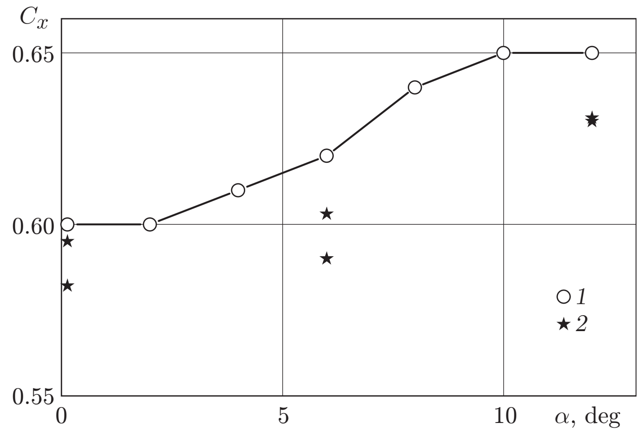

Fig. 5. Drag coefficient C_x of the HB-2 model for different values of α: points 1 refer to the data of ONERA (M $= 10$ and $\mathrm{Re}_L = 2.7 \cdot 10^6$) and points 2 refer to the experimental results obtained in AT-303 wind tunnel (M $= 9.99$ and $\mathrm{Re}_L = 1.7 \cdot 10^6$).

Figure 4 shows the measured longitudinal force f_x and the corresponding drag coefficient C_x as functions of time (α is the angle of attack). To compare these results with data obtained in long-duration tunnels, the data were averaged in time. Figure 5 shows the drag coefficient obtained with allowance for the correction for flow conicity by the Newton theory (Re_L is the Reynolds number based on the model length L) and the data obtained at ONERA (French Aeronautics and Space Research Center). in a steady flow in a wind tunnel with a contoured nozzle (the error in these data was approximately 3%) [6]. The difference in results is about 3.5%.

Thus, the results of solving the model problem and the results of testing the aerodynamic characteristics of the HB-2 reference model on the basis of experimental data obtained in the AT-303 short-duration wind tunnel based at ITAM testify that the method proposed yields reliable solutions.

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